# Vector Geometry

Script for teaching and/or self-organized learning

Theory and Exercises including solutions



Wirth / Siegerist

1st Edition ©2011 Karl Wirth, Zürich Printed in Switzerland

ISBN 978-3-905681-68-0

Orders:

FO-Publishing GmbH Gewerbestrasse 18 CH-8132 Egg/ZH

Telefon++41 44 986 35 70Telefax++41 44 986 35 71Internetfo-publishing.ch/shop

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Karl Wirth, Fritz Siegerist

English translation of the original German edition 'Vektorgeometrie': Mathias Hausherr in cooperation with Karl Wirth

### PREFACE

The script 'Vector Geometry' is released in its 1st edition in English. This new edition corresponds to the the German original 'Vektorgeometrie'.

Many thanks go to Magdalena Fröhlich-Zbinden for the careful proofreading of the English manuscript. Barbara Flütsch published the complete solutions of all exercises online (German language): http://sos-mathe.ch/verz\_v.html. Sincere thanks are also given to her at this point.

#### Contents of the script

The script includes spacial vector geometry as normally treated in a basic course at the gymnasium and pursues the following concept:

- The theory is clearly structured.
- Each page of theory is followed by a page with space for individual notes.
- The exercises are supplemented with solutions, mostly with solution process.

The theory part contains the chapters A to G according to the table of contents. The theory is aimed to quickly arrive at substantial topics, but at the same time to restrict them to the essential. The parametric equation of a plane, for example, is not discussed in the theory part.

The exercise part includes 100 exercises to the individual chapters of the theory. In addition, there are 27 exercises covering the topics of the whole script; their difficulty level corresponds to that of a final exam. Some further exercises right at the end address topics which are not treated in the theory part, including the already mentioned parametric equation of a plane, the sphere equation and the scalar triple product.

#### Use of the script

All theory pages of the chapters (except A) show small gaps (marked with dots for text or formulas and with frames for sketches). These gaps shall keep the reader active while reading the text. Usually the context makes clear what fills the gaps; the correctness can be checked by means of the **appendix**.

At the end of each chapter from C onwards some unsolved exercises are given (tagged with a bold line at the left). On the one hand, they are meant to delve into the theory, on the other hand, they should make familiar with some problems which occur in the exercise part. Therefore, all these exercises should be solved carefully (empty pages exist). The results are listed on the last page of the **appendix**.

According to my understanding, this script can be used to a large extent independently from any teaching or learning method.

Karl Wirth, April 2011

P.S. For suggestions of all kinds please contact me via email: wirthk@gmx.ch

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