Chapter E

47. $\vec{a} = \begin{pmatrix} x \\ 1 \\ 2 \end{pmatrix}$, $\vec{b} = \begin{pmatrix} 2 \\ 0 \\ 1 \end{pmatrix}$

For which x is the area of the parallelogram spanned by the vectors $\vec{a}$ and $\vec{b}$ equal to 3?

48. $A = (8/5/7)$, $B = (5/5/4)$, $M = (4/3/2)$

For the parallelogram $ABCD$ with midpoint M compute

a. the area I, b. the height h_{AB}.

49. $A = (3/5/5)$, $B = (1/1/1)$, $C = (5/3/-3)$

$ABCD$ is the base of a regular square pyramid with the height $h = 9$. Determine the two possible apexes S.

50. $A = (10/0/0)$, $B = (0/6/0)$, $C = (0/0/4)$

The points A, B and C and the origin are the vertexes of a tetrahedron. Compute for this tetrahedron

a. the surface area S,

b. the length h of the height which does not coincide with an edge.

51. $A = (0/10/4)$, $B = (2/14/8)$

For which points C on the x-axis does the triangle ABC have an area of 18?

52. $A = (6/8/3)$, $B = (3/2/1)$, $C = (9/0/-2)$

The points A, B and C are the vertexes of the base of a right three-sided prism with volume 343. The corresponding vertexes of the top are D, E and F, respectively. Determine these vertexes.

53. $P = (0/0/1)$, $Q = (-3/4/3)$, $R = (5/-3/5)$

The points P, Q and R are the centers of three spheres, each with radius 3. There are two planes which touch all three spheres in such a way that the sphere centers lie on the same side of the plane.

a. Determine the tangent points of the sphere with center P.

b. How much is the area I of the triangle formed by the three tangent points of one plane?

Additionally: How many planes touching all three spheres exist?

54. Let be $\vec{a} \neq \vec{o}$ and $\vec{b} \neq \vec{o}$.

a. For which vectors is $|\vec{a} \times \vec{b}| = |\vec{a}||\vec{b}|$?

b. For which vectors is $\vec{a} \times \vec{b} = \vec{o}$?

c. Does the assosciative law $(\vec{a} \times \vec{b}) \times \vec{c} = \vec{a} \times (\vec{b} \times \vec{c})$ hold? Justify!

55. How can one decide, by using the scalar and vector product, whether four points are part of a plane? (cf. with continuing exercise 5, page 52.) How about the following points?

a. $A = (2/1/4)$, $B = (3/2/1)$, $C = (4/3/0)$, $D = (1/0/0)$

b. $A = (-1/1/5)$, $B = (0/2/6)$, $C = (0/1/3)$, $D = (4/3/2)$

56. a. Based on the figure aside, it can be seen geometrically that the three vectors $\vec{b}$, $\vec{c}$ and $\vec{a} \times (\vec{b} \times \vec{c})$ are coplanar. The exact relation is the following:

$$\vec{a} \times (\vec{b} \times \vec{c}) = (\vec{a} \cdot \vec{c})\,\vec{b} - (\vec{a} \cdot \vec{b})\,\vec{c}.$$

b. Show algebraically: This relation is true for any $\vec{a}$, $\vec{b}$ and $\vec{c}$.